Shortest Path First

Dijkstra's Famous Algorithm

"The question of whether computers can think is like the question of whether submarines can swim"

Edsger Wybe Dijkstra

Dijkstra's SP Algorithm

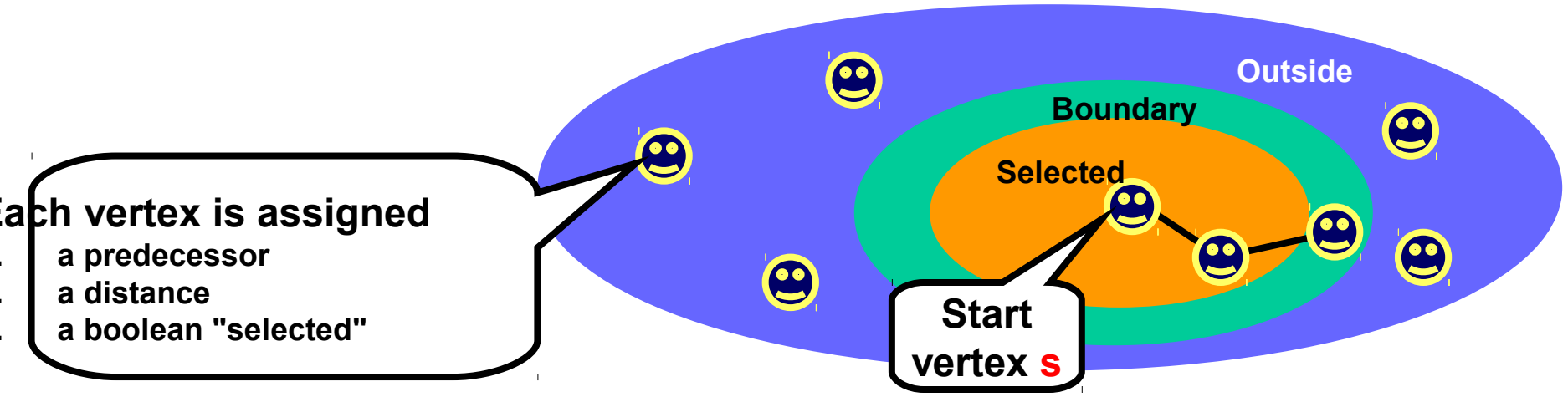
- Famous paper "A note on two problems in connection with graphs" (1959)
- Single source SP problem in a directed graph
- Important applications include
  - Network routing protocols (OSPF, IS-IS)
  - Traveller‘s route planner

Terms

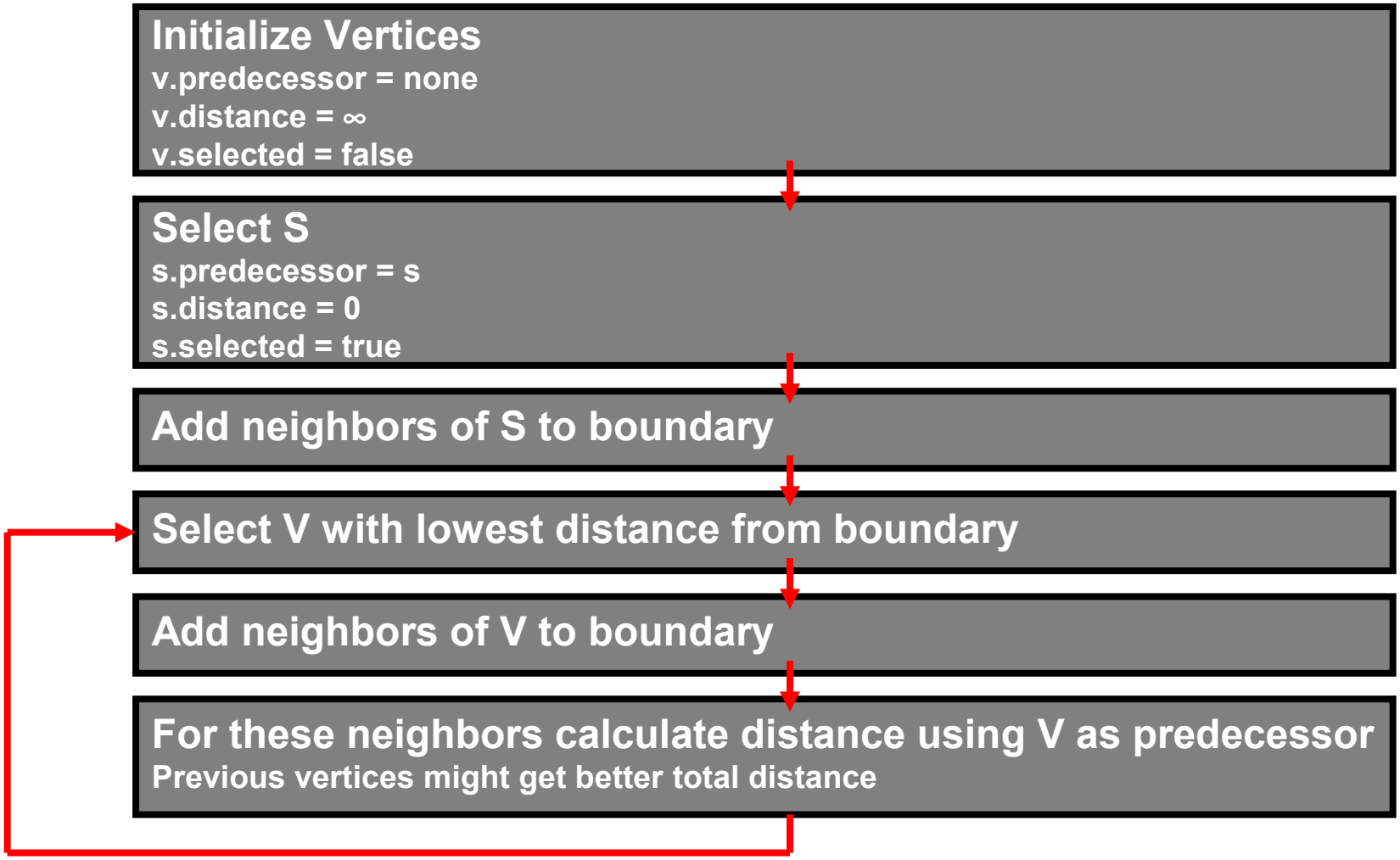
- Graph G(V,E) consists of vertices V and edges E
- Edges are assigned costs c
- "Length" of graph c(G) = sum of all costs
  - Assumed to be positive ("Distance Graph")
- "Distance" between two vertices d(v,v') = min{c(p)}, p…path
  - Can be infinite
- p with c(p) = d(v,v') is called shortest path sp(v,v')

Definitions

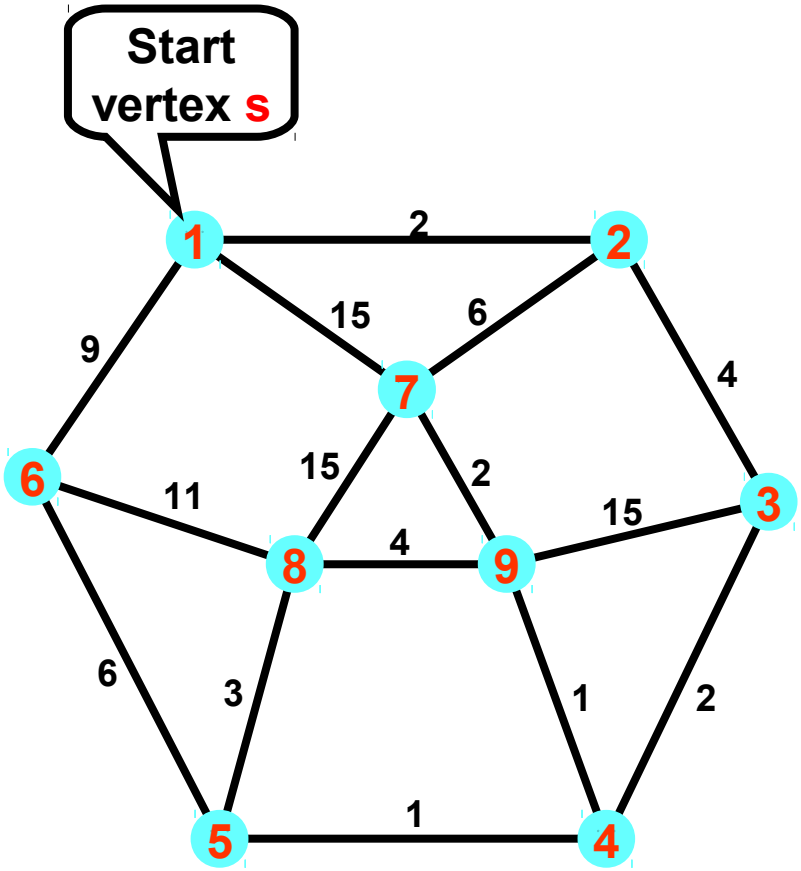
- Select start vertex s
- Three sets of vertices:
  - Selected (sp already calculated)
  - Boundary (currently subject of calculation)
  - Outside (not yet examined)

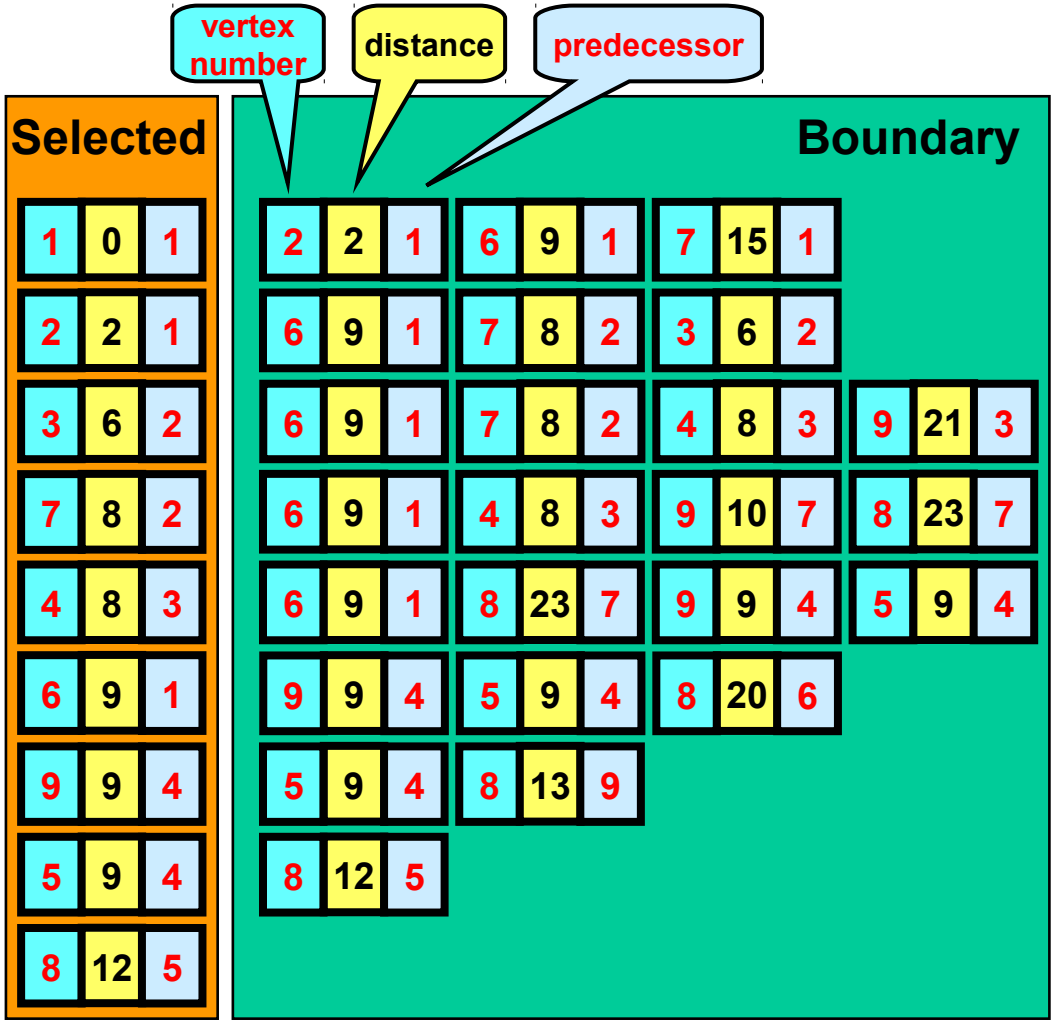


The Algorithm

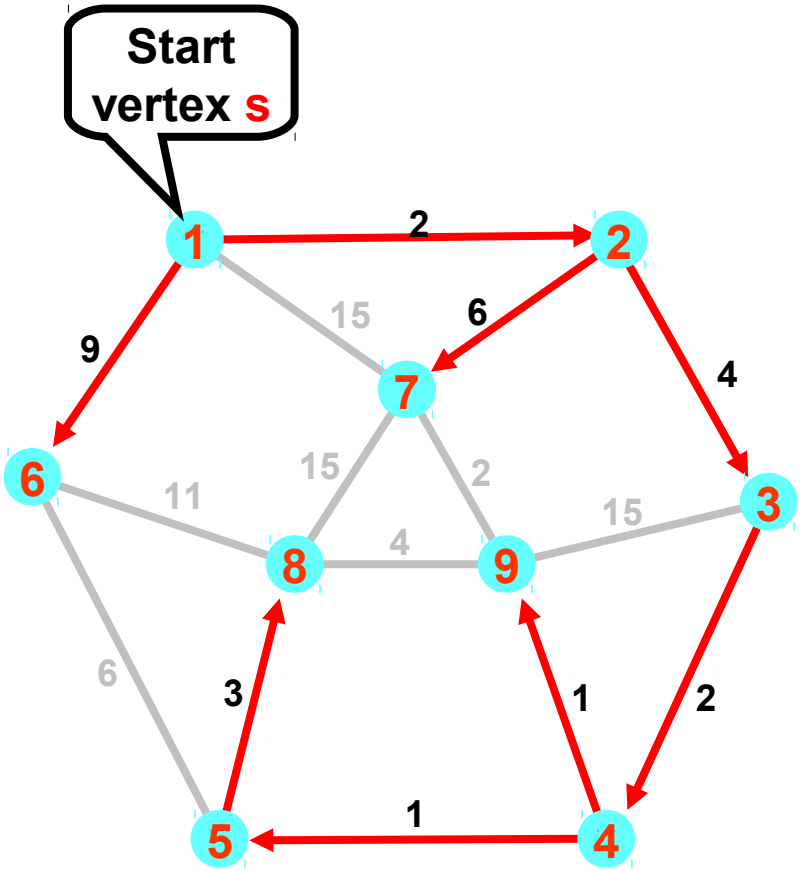


Example


Start vertex s
1
2
3
4
5
6
7
8
9
2
15
6
9
4
15
2
11
15
4
6
3
1
2
1


vertex number
distance
predecessor
Selected
Boundary
1 0 1
2 2 1
3 6 2
7 8 2
4 8 3
6 9 1
9 9 4
5 9 4
8 12 5
2 2 1
6 9 1
7 8 2
3 6 2
6 9 1
4 8 3
9 10 7
8 23 7
6 9 1
8 23 7
9 9 4
5 9 4
9 9 4
5 9 4
8 20 6
5 9 4
8 13 9
8 12 5
6 9 1
7 15 1
9 21 3

Result



Selected		
1	0	1
2	2	1
3	6	2
7	8	2
4	8	3
6	9	1
9	9	4
5	9	4
8	12	5

- Single source SP
- Minimal length
- Complete

Performance

- Greedy algorithm
- Most critical: Implementation of boundary data structure
  - No explicit structure: $O(|V|^2)$
  - Fibonacci heap: $O(|E|+|V| \log |V|)$
- Alternatives
  - Bellman-Ford (RIP) algorithm
  - Floyd-Warshall algorithm
  - A* algorithm
    - Extends SPF with a estimation function to enhance performance in certain situations

About E. W. Dijkstra

- Born in 1930 in Rotterdam
- Degrees in mathematics and theoretical physics from the University of Leyden and a Ph.D. in computing science from the University of Amsterdam
  - Programmer at the Mathematisch Centrum, Amsterdam, 1952-62
  - Professor of mathematics, Eindhoven University of Technology, 1962-1984
  - Burroughs Corporation research fellow, 1973-1984
  - Schlumberger Centennial Chair in Computing Sciences at the University of Texas at Austin, 1984-1999
  - Retired as Professor Emeritus in 1999
  - 1972 recipient of the ACM Turing Award, often viewed as the Nobel Prize for computing
- Died 6 August 2002

Edsger W. Dijkstra
(1930-2002)